Analysis: theoretical expansion of steam compared to ideal gas

Recently I read an experimental analysis of an engine. http://www.modeleng.org/articles/hall01.pdf Making steam consumption and efficiency comparisons of superheated steam v saturated steam and common inlet - exhaust valve v separate inlet and exhaust valve. In that analysis there were charts showing the actual steam pressure in the engine. Ideal gas expansion lines were also included. That perked my interest. The author W. B. Hall did some great work in collecting and processing the experimental data. The analysis presented here is the ideal isentropic expansion curves of steam compared to the ideal gas formula using n=1.35 and n=1.30.

First is to set up approximate expansion parameters to those used in the Hall analysis: From the charts the cutoff pressure and ending pressures were approximated. I then figured the expansion ratio producing the ending pressure.

The first analysis is for saturated steam:

The approximated cutoff parameters. $P_c := 40$ from Fig 4 effects of superheat 0^O F superheat.

$Pa := 14.696$ Atmospheric pressure. PSIG = PSIA - Pa

$Pt_c := ST\_ptdata\left(P_c + Pa, 1, -1, 1\right)$ cutoff steam properties state point saturated state.

$Pt_{c_1} = 286.722$ cutoff temperature saturated steam temp at P_c

$xr := 1.8435$ expansion ratio

$V_c := Pt_{c_2}$ specific volume at cutoff $V_c = 7.826\ ft^3$/lb

$V_e := Pt_{c_2} \cdot xr$ specific volume at end of expansion $V_e = 14.426\ ft^3$/lb

$Pig(v, n) := \left(P_c + Pa\right) \cdot \left(\frac{V_c}{v}\right)^n - Pa$ Ideal gas formula for gage pressure given specific volume and k.
initial pressure gage $P_c = 40$ PSIA $\left(P_c + Pa\right) = 54.696$

Set up to produce plot points. The number of points is 1 + steps set below. The iteration variable i is used to generate a volume array v. The volume array values are then used to generate the corresponding pressure arrays Pst, Pig135 and Pig130. The 135 and 130 corresponding to the k factor of 1.35 and 1.30.

$steps := 200 \qquad i := 0 .. steps \qquad step := \frac{V_c \cdot (xr - 1)}{steps} \qquad v_i := V_c + i \cdot step$

$p := 30$ initial pressure guess needed for the root function. The variable p has no significance other in the root solving of the steam property point below. MathCad root function require and initial guess.

$Pst_i := root\left(ST\_ptdata\left(p, Pt_{c_5}, 5, 1\right)_2 - v_i, p\right) - Pa$ Generate the PSIG line of isentropic expansion.

$Pig130_i := Pig\left(v_i, 1.30\right)$

$Pig135_i := Pig\left(v_i, 1.35\right)$

$Pt_{shc} := ST\_ptdata\left(P_c + Pa, Pt_{c_1} + 93.1, 1, 1\right)$ cutoff steam properties state point super heated state.

$V_e := Pt_{shc_2} \cdot xr \qquad step := \frac{Pt_{shc_2} \cdot (xr - 1)}{steps} \qquad v_i := Pt_{shc_2} + i \cdot step \qquad Pt_{shc_2} = 8.947 \qquad V_e = 16.494$

$Pht_i := root\left(ST\_ptdata\left(p, Pt_{shc_5}, 5, 1\right)_2 - v_i, p\right) - Pa$ Generate the PSIG line of isentropic expansion.

After doing the above I saw some interesting facts of saturated steam expansion: My theoritical expansion lines showed simular results with Hall's saturated steam line except for the closeness through the first part of expansion. I figures that was due to throttling. The saturated steam may have become slightly super heated droping to the 42 PSI from

$\text{inlet} := \text{ST\_ptdata}(60 + \text{Pa}, 1, -1, 1)\ \text{cutoff} := \text{ST\_ptdata}\left(P_c + \text{Pa}, \text{inlet}_4, 4, 1\right)\text{off}_1 = 297.854 \quad \text{cutoff}_1 - \text{Pt}_{c_1} = 11.132$ uper heat

$V_e := \text{cutoff}_2 \cdot \text{xr} \quad \text{step} := \dfrac{\text{cutoff}_2 \cdot (\text{xr} - 1)}{\text{steps}} \qquad v_i := \text{cutoff}_2 + i \cdot \text{step} \qquad \text{cutoff}_2 = 7.964 \qquad V_e = 14.681$

$\text{Pth}_i := \text{root}\left(\text{ST\_ptdata}\left(p, \text{cutoff}_5, 5, 1\right)2 - v_i, p\right) - \text{Pa}$ Generate the pressure line of isentropic steam expansion.

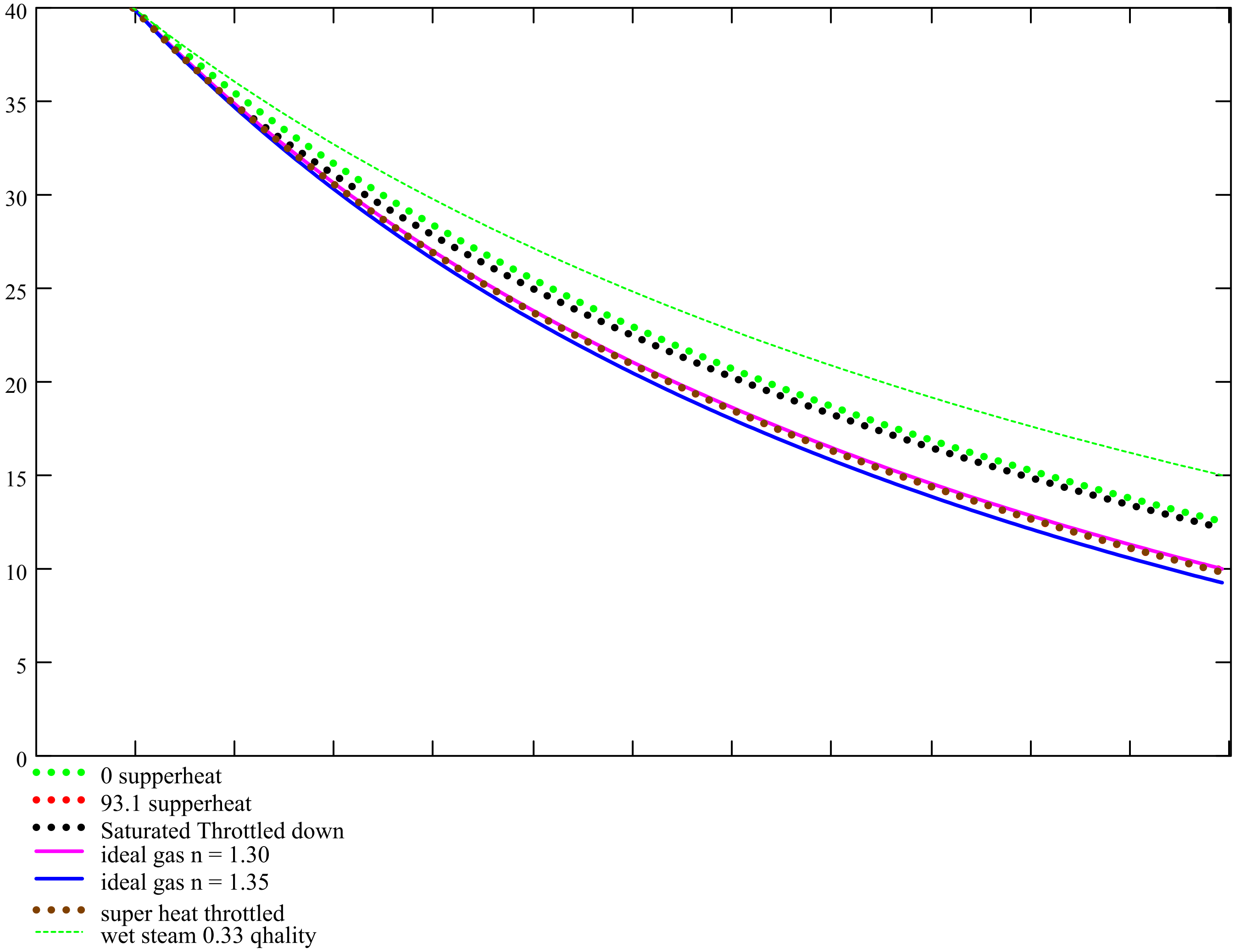
$\text{super} := \text{ST\_ptdata}\left(60 + \text{Pa}, \text{inlet}_1 + 93.1, 1, 1\right) \qquad \text{super}_c := \text{ST\_ptdata}\left(P_c + \text{Pa}, \text{super}_4, 4, 1\right) \qquad \text{super}_{c_1} - \text{Pt}_{c_1} = 107.765$

$\text{step} := \dfrac{\text{super}_{c_2} \cdot (\text{xr} - 1)}{\text{steps}} \qquad v_i := \text{super}_{c_2} + i \cdot \text{step}$

$\text{Psth}_i := \text{root}\left(\text{ST\_ptdata}\left(p, \text{super}_{c_5}, 5, 1\right)2 - v_i, p\right) - \text{Pa}$ Generate the pressure line

$\text{Pw}_c := \text{ST\_ptdata}\left(P_c + \text{Pa}, 0.33, -1, 1\right) \qquad \text{step} := \dfrac{\text{Pw}_{c_2} \cdot (\text{xr} - 1)}{\text{steps}} \qquad v_i := \text{Pw}_{c_2} + i \cdot \text{step}$ One more line of high water content.

$\text{Pw}_i := \text{root}\left(\text{ST\_ptdata}\left(p, \text{Pw}_{c_5}, 5, 1\right)2 - v_i, p\right) - \text{Pa}$ Generate the PSIG line of isentropic expansion.



Calculate ending steam properties

$Pw_e := \mathrm{ST\_ptdata}\left(Pw_{steps} + Pa, Pw_{c_5}, 5, 1\right)$

$Pst_e := \mathrm{ST\_ptdata}\left(Pst_{steps} + Pa, Pt_{c_5}, 5, 1\right)$

$Pth_e := \mathrm{ST\_ptdata}\left(Pth_{steps} + Pa, cutoff_5, 5, 1\right)$

$Pht_e := \mathrm{ST\_ptdata}\left(Pht_{steps} + Pa, Pt_{shc_5}, 5, 1\right)$

$Psth_e := \mathrm{ST\_ptdata}\left(Psth_{steps} + Pa, super_{c_5}, 5, 1\right)$

pressure	**initial condition**	**calculate ideal cycle work BTU/lb**	**ending quality**
$Pw_{steps} = 15$	saturated steam at 40 PSIG	$Pw_{c_4} - Pw_{e_4} = 16.064$	$Pw_{e_6} = 0.344$
$Pst_{steps} = 12.531$	saturated steam at 40 PSIG	$Pt_{c_4} - Pst_{e_4} = 52.93$	$Pst_{e_6} = 0.959$
$Pth_{steps} = 12.18$	saturated steam at 60 PSIG	$cutoff_4 - Pth_{e_4} = 54.254$	$Pth_{e_6} = 0.964$
$Pht_{steps} = 9.936$	40 PSIG 93.1 F supper heat	$Pt_{shc_4} - Pht_{e_4} = 65.817$	$Pht_{e_6} = 0.998$
$Psth_{steps} = 9.81$	60 PSIG 93.1 F supperheat	$super_4 - Psth_{e_4} = 67.475$	$Psth_{e_6} = 1$
$Pig130_{steps} = 10$	ideal gas n=1.30		
$Pig135_{steps} = 9.256$	ideal gas n=1.35		

Note the 60 PSIG pressures are throttled to 40 and then expanded.